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Consistent momentum space regularization/renormalization of supersymmetric quantum field theories: the three-loop β-function for the Wess-Zumino model

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Abstract: We compute the three loop β function of the Wess-Zumino model to motivate implicit regularization (IR) as a consistent and practical momentum-space framework to study supersymmetric quantum field theories. In this framework which works essentially in the physical dimension of the theory we show that ultraviolet are clearly disantangled from infrared divergences. We obtain consistent results which motivate the method as a good choice to study supersymmetry anomalies in quantum field theories.

Keywords: Renormalization Regularization and Renormalons, Renormalization Group, Supersymmetry Phenomenology, Superspaces.

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1. Introduction

Dimensional regularization (DR) is a remarkable framework which besides preserving gauge invariance is relatively simple from the computational standpoint. For this reason it has become the stantard method in perturbative calculations in quantum field theory. The regularization and renormalization of supersymmetric gauge theories is, however, a more involved problem. Momentum space subtraction schemes such as BPHZ [1] although supersymmetric are not gauge invariant [2]. DR does not mantain the balance between bosonic and fermionic degrees of freedom due to the analytic continuation on the space-time dimension and therefore breaks supersymmetry. Such breaking demands the calculation of compensating supersymmetry restoring counterterms. A practical useful modification of DR is dimensional reduction (DRed) [3, 4] which, however, can lead to inconsistencies in supersymmetric gauge field theories beyond one loop order. As long as such inconsistencies can be tamed and symmetry restoring counterterms can be unambiguously generated by imposing the validity of the Slavnov-Taylor and Ward identities [5] the use of DR and DRed are obviously justified. However it is not always the case that the symmetry of the lagrangian is still a symmetry of the full quantum effective action. Supersymmetry anomalies can in principle be generated and some erroneous claims about their existence have

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occurred because it is difficult to distinguish between a genuine anomaly and an apparent violation of a supersymmetric Ward identity due to the use of an ill-defined regularization scheme [6]. In the case of a genuine anomaly no symmetry restoring (finite) counterterms can be obtained.

A regularization/renormalization framework that: *a*) does not modify the field theoretical content of the bare lagrangian (and hence does not unnecessarily complicate the Feynman rules); *b*) works in the physical dimension of the model; *c*) preserves gauge invariance and *d*) is friendly from the calculational viewpoint, is therefore desirable to tackle the issues that we discussed above. In this sense two relatively new frameworks deserve special attention: Differential and Implicit regularization/renormatization (DfR and IR, respectively).

DfR is a coordinate space method that defines the correlation functions without introducing a regulator or counterterms [7]–[17]. A well defined prescription that (in a minimal sense) extends product of distributions into a distribution automatically delivers renormalized finite amplitudes. The latter contains an arbitrary mass scale which must be introduced by dimensional reasons and plays the role of a renormalization group scale. Gauge invariance may be sistematicaly implemented in a constrained version of DfR, at least to one loop order. Finally contact with momentum space is made by means of Fourier transforms.

On the other hand IR is a momentum space scheme where scattering amplitudes with fixed external momenta are constructed in the first place. In section 2 we briefly outline this method.

The purpose of this paper is to motivate IR as a sound, symmetry preserving and practical framework to study dimension specific theories, among which chiral, topological and supersymmetric gauge theories are of prime interest. We have shown in [19]–[22] how gauge invariance is preserved within IR. Here we use the massless Wess-Zumino model as a testing ground for the consistency of IR in preserving supersymmetry. We present the Feynman diagram calculation of this model in sections 4, 5 and 6. As a nontrivial check we compute in section 7 the β-function to three loop order and verify the agreement with other consistent methods. As a by-product we show how infrared and ultraviolet divergences are clearly disantangled (in opposite to dimensional methods). This is important in the study of certain supersymmetry anomalies. We also define what is meant by a minimal subtraction within IR and compare with DR and DfR. Finally, in section 8, we address some theoretical and phenomenological problems where IR could be useful.

2. Implicit regularization/renormalization

The main idea behind IR is to isolate the divergences from an amplitude as irreducible loop integrals (ILI) which do not depend upon the external momenta by judiciously using the identity:

$$\frac{1}{[(k+k_i)^2-m^2]}=\sum_{j=0}^{N}\frac{(-1)^j(k_i^2+2k_i\cdot k)^j}{(k^2-m^2)^{j+1}[(k+k_i)^2-m^2]^j}\,. \tag{2.1}$$

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In the equation above, k_i are the external momenta and N is chosen so that the last term is finite under integration over k. For instance take the logarithmically divergent integral in four dimensions

$$\Gamma(p^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)[(k+p)^2-m^2]} \,. \tag{2.2}$$

We use (2.1) with $N=1$ to write

$$\Gamma(p^2) = \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)^2} - \int \frac{d^4k}{(2\pi)^4} \frac{p^2+2p\cdot k}{(k^2-m^2)^2[(k+p)^2-m^2]} \,. \tag{2.3}$$

Note that the second term on the rhs of the equation above is finite whereas the first is an ILI, namely,

$$I_{\log}(m^2) \equiv \int \frac{d^4k}{(2\pi)^4} \frac{1}{(k^2-m^2)^2} \,, \tag{2.4}$$

which characterizes the ultraviolet behaviour of the amplitude and need not to be explicitly evaluated. They can be fully absorbed in the definition of the renormalization constants. In order to define a mass independent scheme (or in the case of massless theories) we use the identity (4.6) which we introduce in section 4. Such identity introduces naturally an arbitrary scale which plays the role of renormalization group scale. Infinities of higher order are equally displayed as ILI such as $I_{\rm quad}(m^2)$, $I_{\rm lin}(m^2)$ etc., for quadratic and linearly divergent integrals, respectively. Local arbitrary counterterms can show up in IR as (finite) differences between divergent integrals of the same superficial degree of divergence which can sistematically be cast into a set of "consistency relations" according to the space-time dimension. Such relations can be differences between logarithmically, linearly or even quadratically divergent loop integrals whose value may be fixed on physical grounds [20]. At the one loop level they are related to momentum routing invariance in the loop of a Feynman diagram. A constrained version of IR corresponds to setting such consistency relations to vanish. In this way, abelian and nonabelian gauge invariance can be shown to automatically implemented. Ultimately, in a more general context, such local arbitrary counterterms parametrized by the consistency relations should be left arbitrary till the end of the calculation when the symmetry content of the underlying model may fix its value. We address the reader to ref. [18]–[24] for details and applications of IR.

Overlapping divergences which are the chief complication in any renormalization scheme can also be handled in a schematic way within IR. This was illustrated to n-loop order [25] in the context of ϕ^3 theory in 6 dimensions.

3. Perturbation expansion of the Wess-Zumino model

We closely follow the notation and the conventions of ref. [26]. The Wess-Zumino (WZ) model superspace action reads

$$\mathcal{S} = \int d^4x \left\{ \int d^2\theta d^2\bar\theta \bar\phi_0\phi_0 - \frac{g_0}{3!}\left(\int d^2\theta \phi_0^3 - \int d^2\bar\theta \bar\phi_0^3 \right) \right\} , \tag{3.1}$$

where $(x^a, \theta^\alpha, \bar\theta^{\dot\alpha})$, $a = 1,2,3,4$, $\alpha = +-$, $\dot\alpha = \dot+, \dot-$ are coordinates of $d=4$, $N=1$ superspace and ϕ ($\bar\phi$) is a chiral (antichiral) superfield.

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The lagrangian can be written in terms of component fields

$$\phi_0 = \exp\left(i\theta^\alpha\bar{\theta}^{\dot{\beta}}\ \sigma^a_{\alpha\dot{\beta}}\ \partial_a\right)\left(\frac{1}{\sqrt{2}}(A+iB)+\theta^\alpha\psi_\alpha+\frac{1}{\sqrt{2}}(F-iG)\theta^2\right), \tag{3.2}$$

which after eliminating the auxiliary fields F, G, will have the form

$$\begin{aligned}\mathcal{L} = &-\frac{1}{2}(\partial_\mu A)^2-\frac{1}{2}(\partial_\mu B)^2-\frac{1}{2}\bar{\psi}\gamma\cdot\partial\psi- \\ &-\frac{1}{16}g_0^2(A^2+B^2)^2+\frac{g_0}{2\sqrt{2}}\bar{\psi}(A+i\gamma_5 B)\psi\,.\end{aligned} \tag{3.3}$$

The WZ model involves only one renormalization constant since only propagator-type diagrams can diverge. To see that, one may employ the superfield power counting rules described in [27] to conclude that the three-point function is finite. Defining the renormalization constants as

$$\begin{aligned}g_0 &= Z_g g \\ \phi_0 &= Z_\phi^{1/2}\phi\,,\end{aligned} \tag{3.4}$$

and using that $g_0\phi_0^3$ is finite (and hence $Z_g Z_\phi^{3/2}=1$) enables us to write

$$g_0 = Z_\phi^{-3/2}g\,. \tag{3.5}$$

Thus for computing the β-function it is sufficient to calculate Z_ϕ (i.e. the divergent structure of the two-point function). We expand Z_ϕ into a power series in the coupling constant,

$$Z_\phi = 1+\left(\frac{g}{4\pi}\right)^2 Z_1+\left(\frac{g}{4\pi}\right)^4 Z_2+\left(\frac{g}{4\pi}\right)^6 Z_3+\cdots \tag{3.6}$$

and write the part of the effective action which is linear in ϕ and $\bar{\phi}$ like

$$\Gamma_2(\phi,\bar{\phi}) = -i\int_{k,\theta,\bar{\theta}}\bar{\phi}(-k,\theta,\bar{\theta})\phi(k,\theta,\bar{\theta})\Delta^{-1}(k)\,, \tag{3.7}$$

where $\int_k$ and $\int_\theta$ stand for $\int d^4k/(2\pi)^4$ and $\int d^2\theta$, respectively. Following [26] we notice that in the renormalized lagrangian Z_ϕ appears only in the kinetic term $\bar{\phi}_0\phi_0 = Z_\phi\bar{\phi}\phi$ which amounts to introducing a factor of Z_ϕ^{-1} for each propagator in the diagrams. Hence we can write

$$\begin{aligned}\Delta^{-1}(p) = &Z_\phi+\frac{1}{2}\left(\frac{g}{4\pi}\right)^2 Z_\phi^{-2}F(p^2)+\frac{1}{2}\left(\frac{g}{4\pi}\right)^4 Z_\phi^{-5}G(p^2)+ \\ &+\left(\frac{g}{4\pi}\right)^6 Z_\phi^{-8}H(p^2)+\cdots\,,\end{aligned} \tag{3.8}$$

in which $\frac{1}{2}g^2/(4\pi)^2F(p^2)$ represents the contribution of the one loop diagram (figure 1), $\frac{1}{2}g^4/(4\pi)^4G(p^2)$ represents the two loop contribution (figure 2), $\frac{1}{2}$ being a symmetry factor, and $g/(4\pi)^6H(p^2)$ refers to the three loop diagrams (figures 3 to 6). Now we take equation

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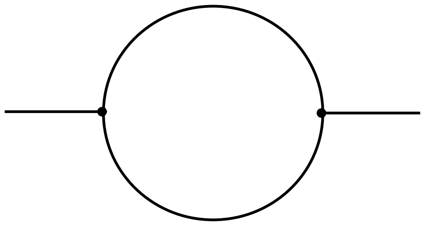

Figure 1: $F(p^2)$.

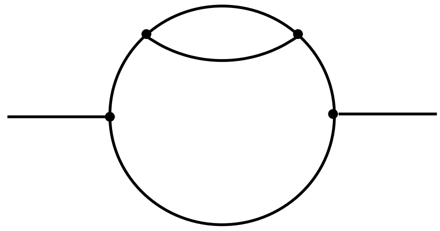

Figure 2: $G(p^2)$.

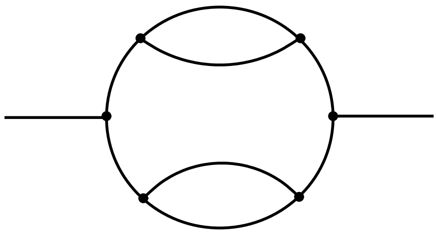

Figure 3: $H_3(p^2)$.

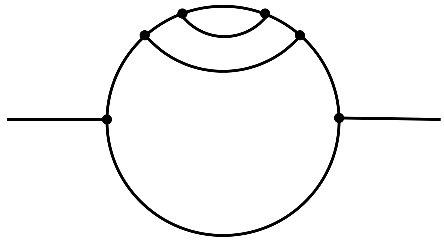

Figure 4: $H_4(p^2)$.

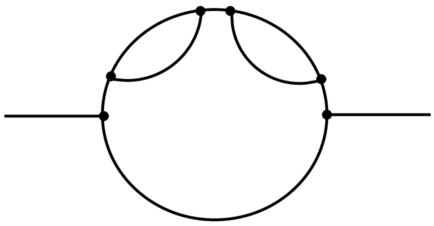

Figure 5: $H_5(p^2)$.

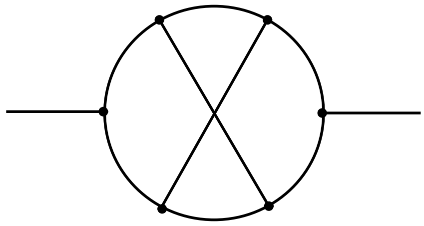

Figure 6: $H_6(p^2)$.

(3.6) in equation (3.8) and reorganize the power expansion to write

$$\begin{aligned}\Gamma_2(\phi,\bar{\phi}) = &-i\int_{p,\theta,\bar{\theta}} \bar{\phi}(-p,\theta,\bar{\theta})\phi(p,\theta,\bar{\theta}) \times \\ &\times\left\{ Z_\phi + \frac{1}{2}\left(\frac{g}{4\pi}\right)^2 F(p^2) + \left(\frac{g}{4\pi}\right)^4 \left[\frac{1}{2}G(p^2) - Z_1 F(p^2)\right] + \right. \\ &\left. + \left(\frac{g}{4\pi}\right)^6 \left[H(p^2) - \frac{5}{2}Z_1 G(p^2) - \left(Z_2 - \frac{3}{2}Z_1^2\right) F(p^2)\right] + O\Big(\left(\frac{g}{4\pi}\right)^8\Big)\right\}.\end{aligned} \tag{3.9}$$

The Feynman rules for the Wess-Zumino model are well-known (please see [26, 28]) so we shall not derive them here. The physical interpretation of equation (3.9) is straightforward. It defines the renormalization constants Z_i's at each loop order after the infinities corresponding to subgraphs of previous orders are duly subtracted. Diagrammatically it amounts to substituting the divergent subdiagrams of a diagram with their finite part order by order, as we shall see in the next sections. We shall perform such substitution after subtracting the divergences in a minimal sense within our approach, namely by subtracting irreducible loop integrals.

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4. One loop contribution

The one loop contribution to the propagator correction is represented by the diagram depicted in figure 1. Application of the Feynman rules to this diagram yields [26, 27]:

$$\Gamma_2^{(1)} = (-i)^2 g^2 \int_p d^4\theta\, \bar{\phi}(-p,\theta)\phi(p,\theta) \int_k \frac{1}{k^2(p+k)^2}\,. \tag{4.1}$$

From the equation above and (3.9) we identify

$$F(p^2) = -i(4\pi)^2 \int_k \frac{1}{(k^2-\mu^2)[(p+k)^2-\mu^2]} \tag{4.2}$$

in which we have introduced an infrared cutoff μ. Following our approach, an irreducible loop integral can be separated from the amplitude above with the help of equation (2.1) to give

$$\begin{aligned} F(p^2) &= -i(4\pi)^2 \left(\int_k \frac{1}{(k^2-\mu^2)^2} - \int_k \frac{p^2+2p\cdot k}{(k^2-\mu^2)^2[(p+k)^2-\mu^2]}\right) && (4.3)\\ &\equiv -i(4\pi)^2 \left(I_{\log}(\mu^2) - b\int_0^1 dz\, \ln\left(\frac{p^2 z(z-1)}{\mu^2}+1\right)\right), && (4.4)\end{aligned}$$

where, henceforth we define

$$b \equiv \frac{i}{(4\pi)^2} \tag{4.5}$$

and $I_{\log}(\mu^2)$ is given by equation (2.4). Since the limit where $\mu \to 0$ is infrared ill-defined in $I_{\log}(\mu^2)$, the correct ultraviolet behaviour is obtained by exchanging the latter against $I_{\log}(\lambda^2)$ provided $\lambda^2 \neq 0$ by means of the identity

$$I_{\log}(\mu^2) = I_{\log}(\lambda^2) + b\ln\left(\frac{\lambda^2}{\mu^2}\right). \tag{4.6}$$

Hence (4.6) splits the ultraviolet and infrared divergences and as a byproduct it parametrizes the arbitrariness in separating the divergent from the finite part. That is because the infrared divergent piece in the right hand side of (4.6) cancels out against another piece coming from the (ultraviolet) finite part of the amplitude which may be easily integrated to give

$$F(p^2) = -i(4\pi)^2 \left(I_{\log}(\lambda^2) + b\ln\left(\frac{-\lambda^2 e^2}{p^2}\right)\right). \tag{4.7}$$

Consequently λ is the natural candidate for a renormalization scale from which we may construct a Callan-Symanzik renormalization group equation. A minimal subtraction within our approach amounts to defining, with the help of (3.9),

$$Z_1 = \frac{i}{2}(4\pi)^2 I_{\log}(\lambda^2)\,, \qquad \lambda^2 \neq 0\,, \tag{4.8}$$

i.e., we have subtracted only the irreducible loop integral. For future reference, we define the finite part of $F(p^2)$ as

$$F_{\rm fin} = -i(4\pi)^2 b\ln\left(\frac{-\lambda^2 e^2}{p^2}\right), \tag{4.9}$$

where λ plays the role of an arbitrary local counterterm. Moreover $F_{\rm fin}$ satisfies a Callan-Symanzik renormalization group equation with scale λ [22].

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For a massive theory a minimal mass independent scheme is defined in a similar fashion. In [22] we have compared our minimal subtraction scheme with the MS scheme in dimensional renormalization as well as in differential renormalization. The arbitrary scales appearing in each framework are related to each other as we shall discuss in section 7 (see also [29]).

5. Two loop contribution

The propagator receives just one two-loop contribution (figure 2) because the only propagator is the one from ϕ to $\bar{\phi}$ (there is no $\phi\phi$ or $\bar{\phi}\bar{\phi}$ propagators). The corresponding amplitude can be constructed by direct application of the Feynman rules. After some straightforward algebra it can be reduced to

$$\Gamma_2^{(2)} = (-i)^3 g^4 \int_p d^4\theta\, \bar{\phi}(-p,\theta)\phi(p,\theta)(-1) \int_{q,k} \frac{1}{k^2 q^2 (k+q)^2 (p+q)^2}\,, \tag{5.1}$$

which enables us to identify

$$G(p^2) = (4\pi)^4 \int_q \frac{1}{q^2(p+q)^2} \int_k \frac{1}{k^2(k+q)^2}\,. \tag{5.2}$$

Let us have a closer look at the $O(g^4)$ term in equation (3.9) viz.,

$$\left(\frac{g}{4\pi}\right)^4 \left[Z_2 + \frac{1}{2}G(p^2) - Z_1 F(p^2)\right], \tag{5.3}$$

which graphically corresponds to figure 2. Perturbative renormalization is inductive in the definition of the counterterms. Thus in defining Z_2 we firstly ought to subtract the subdivergence through Z_1 which was defined in the previous order as prescribed in (5.3). We can explicitly evaluate $1/2\, G(p^2) - Z_1 F(p^2)$ using (4.7), (4.8) and (5.2) to conclude that such operation amounts to substituting the subintegration over k in (5.2) with $F_{\rm fin}(p^2)$ divided by $-i(4\pi)^2$ to define:

$$\begin{aligned}\widetilde{G}(p^2) &= b(4\pi)^4 \int_q \frac{1}{q^2(p+q)^2} \ln\left(\frac{-\lambda^2 e^2}{q^2}\right) \\ &= \frac{1}{2}G(p^2) - Z_1 F(p^2)\,.\end{aligned} \tag{5.4}$$

Hereforth the tilde means that we have subtracted all the subdivergencies. The graphical interpretation of such procedure is clear.

We proceed to define the renormalization constant Z_2. Introducing an infrared cutoff in (5.4) enables us to write

$$\widetilde{G}(p^2) = b(4\pi)^4 \int_q \frac{1}{(q^2-\mu^2)[(p+q)^2-\mu^2]} \ln\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right). \tag{5.5}$$

The irreducible loop integral can be separated just as we did at the one loop level as follows

$$\begin{aligned}\widetilde{G}(p^2) &= b(4\pi)^4 \int_q \frac{1}{(q^2-\mu^2)^2} \ln\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right) - \\ &\quad - b(4\pi)^4 \int_q \frac{p^2 + 2p\cdot q}{(q^2-\mu^2)^2[(p+q)^2-\mu^2]} \ln\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right).\end{aligned} \tag{5.6}$$

Notice that the second term on the right hand side is finite whilst the first term is an ILI, namely

$$\int_q \frac{1}{(q^2-\mu^2)^2} \ln\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right) \equiv I^{(2)}_{\log}(\mu^2)\,. \tag{5.7}$$

Again, a bona fide renormalization constant should be infrared finite. Thus we subtract $I^{(2)}_{\log}(\lambda^2)$ with $\lambda \neq 0$ by parametrizing the infrared divergence by means of the identity

$$I^{(2)}_{\log}(\mu^2) = I^{(2)}_{\log}(\lambda^2) + b\left[\frac{1}{2}\ln^2\left(\frac{\mu^2}{\lambda^2 e^2}\right) - 2\right]. \tag{5.8}$$

Relation (5.8) is the two loop analog of (4.6) . Now taking (5.8) and (5.7) into (5.6) permits us to subtract the divergence by defining the renormalization constant of order g^4 as

$$Z_2 = -\frac{i}{2}(4\pi)^2 I^{(2)}_{\log}(\lambda^2)\,. \tag{5.9}$$

In the next section we shall evaluate the three-loop contributions to the propagator and define the corresponding counterterms. Thus it is convenient to simplify the finite part of the two loop contribution as there will be two loop subgraphs at three-loop order. We show in appendix A that

$$\begin{aligned} G_{\text{fin}}(p^2) &= \widetilde{G}_{\text{fin}}(p^2) \\ &= -\frac{1}{2}\ln^2\left(\frac{-\lambda^2 e^2}{p^2}\right) - \ln\left(\frac{-\lambda^2 e^2}{p^2}\right). \end{aligned} \tag{5.10}$$

It is worthwhile mentioning that $G_{\text{fin}}(p^2)$ is infrared safe as it should, since the limit where $\mu \to 0$ is well defined through a cancellation of terms in equations (5.6) and (5.8) .

6. Three loop contributions

The diagrams depicted in figures 3 to 6 represent the three loop order contributions to the propagator. Feynman rules can be directly applied to give [26] :

$$\Gamma_2^{(3)} = (-i)^4 g^6 \int_p d^4\theta \bar{\phi}(-p,\theta)\phi(p,\theta)\frac{-i}{(4\pi)^6}\left(H_3(p^2)+H_4(p^2)+H_5(p^2)+H_6(p^2)\right), \tag{6.1}$$

$H_3 \ldots H_6$ being the result of integrating over the δ functions of the superspace coordinates and eliminating the covariant derivatives. The factor $-i/(4\pi)^6$ appears so that the H_i's here agree with the definition expressed in equation (3.9). In the latter H stands for $H_3 + \cdots + H_6$.

Take the first three loop diagram (figure 3). It contributes to (6.1) with

$$H_3 = \frac{i(4\pi)^6}{8}\int_q \frac{1}{q^2(p+q)^2}\int_k \frac{1}{k^2(k+p+q)^2}\int_l \frac{1}{l^2(q+l)^2}\,. \tag{6.2}$$

Note that it contains two one-loop subdiagrams. According to what we discussed earlier we can substitute the integral over l with $F_{\text{fin}}(q^2)/[-i(4\pi)^2]$ (equation (4.9)) at once. As for the integral over k , it can be cast with the help of (2.1) as

$$\frac{i(4\pi)^6}{8}\left[\int_q \frac{1}{q^2(p+q)^2}\left(\int_k \frac{1}{k^2(k+q)^2} - \frac{p^2+2p\cdot(k+q)}{k^2(k+q)^2(p+k+q)^2}\right) b\ln\left(\frac{-\lambda^2 e^2}{q^2}\right)\right]. \tag{6.3}$$

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This is useful because doing so we have contributed to free the ultraviolet divergent piece from external momentum dependence. Note that only the first piece of equation (6.3), namely

$$\frac{i(4\pi)^6}{8}\int_q \frac{1}{q^2(p+q)^2}\int_k \frac{1}{k^2(k+q)^2} b\ln\left(\frac{-\lambda^2 e^2}{q^2}\right) \tag{6.4}$$

is ultraviolet divergent. Thus we may promptly substitute the integral over k (which represents a one-loop subdiagram with external momentum q) in (6.4) with $F_{\rm fin}(q^2)/[-i(4\pi)^2]$ to define:

$$\widetilde{H}_3(p^2) = \frac{i(4\pi)^6}{8}\int_q \frac{1}{q^2(p+q)^2}\left[b\ln\left(\frac{-\lambda^2 e^2}{q^2}\right)\right]^2 . \tag{6.5}$$

To define the irreducible loop integral which will contribute to Z_3 we introduce an infrared cutoff μ and remove the external momentum dependence in $\widetilde{H}_3(p^2)$ in a similar fashion to equations (5.5)–(5.6). That is to say

$$\begin{aligned}\widetilde{H}_3(p^2) &= \frac{i(4\pi)^6}{8}\int_q \frac{1}{(q^2-\mu^2)^2} b^2 \ln^2\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right) + +\mathcal{F}_3(p^2,\mu^2)\\ &= -\frac{i(4\pi)^2}{8} I^{(3)}_{\rm log}(\mu^2) + \mathcal{F}_3(p^2,\mu^2)\,,\end{aligned} \tag{6.6}$$

in which we defined another logarithmically divergent ILI,

$$I^{(3)}_{\rm log}(\mu^2) \equiv \int_q \frac{1}{(q^2-\mu^2)^2}\ln^2\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right), \tag{6.7}$$

whereas $\mathcal{F}_3(p^2,\mu^2)$ stands for an ultraviolet finite piece. This diagram will contribute to Z_3 with an infrared finite term

$$Z_3^{(3)} = \frac{i(4\pi)^2}{8} I^{(3)}_{\rm log}(\lambda^2)\,, \tag{6.8}$$

$\lambda \neq 0$ since one can easily verify that

$$I^{(3)}_{\rm log}(\mu^2) = I^{(3)}_{\rm log}(\lambda^2) - b\left[\frac{8}{3} + \frac{1}{3}\ln^3\left(\frac{\mu^2}{\lambda^2 e^2}\right)\right]. \tag{6.9}$$

In analogy with our calculations at the one and two loop orders, the infrared divergent piece in equation above is expected to cancel the infrared divergence in $\mathcal{F}_3(p^2,\mu^2)$ to render a well defined finite part. This is indeed the case as one can prove after some straightforward algebra. Therefore we may write

$$\widetilde{H}_3(p^2) = -\frac{i(4\pi)^2}{8} I^{(3)}_{\rm log}(\lambda^2) + \mathcal{G}_3(p^2,\lambda^2)\,. \tag{6.10}$$

The second three-loop contribution is represented by figure 4 from which one gets

$$H_4 = \frac{i(4\pi)^6}{2}\int_q \frac{1}{q^2(p+q)^2}\int_l \frac{1}{l^2(q+l)^2}\int_k \frac{1}{k^2(k+l)^2}\,. \tag{6.11}$$

One can see from this graph that it contains as a subdiagram the two loop graph shown in figure 2 which is represented by the integrals over l and k in the amplitude above, q playing the role of external momentum. The procedure is identical as we did for H_3 so we shall only summarize the steps. We replace the integrals over l and k by $G_{\rm fin}(q^2)/(4\pi)^4$ (equation (5.10)) and introduce an infrared cutoff, μ. Next we expand the propagator which contains the external momentum p in the usual fashion so to define an irreducible loop integral which is p-independent. We obtain

$$\widetilde{H}_4(p^2) = \frac{-i(4\pi)^6}{2(4\pi)^4} \int_q \frac{1}{(q^2-\mu^2)^2} \left[\frac{1}{2}\ln^2\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right) + \ln\left(\frac{-\lambda^2 e^2}{q^2-\mu^2}\right)\right] + \mathcal{F}_4(p^2,\mu^2)\,, \quad (6.12)$$

$\mathcal{F}_4$ being an ultraviolet finite term. Using (5.7), (5.8), (6.7) and (6.9) allows us to write

$$\widetilde{H}_4(p^2) = \frac{-i(4\pi)^2}{2}\left(\frac{1}{2}I^{(3)}_{\log}(\lambda^2) + I^{(2)}_{\log}(\lambda^2)\right) + \mathcal{G}_4(p^2,\lambda^2)\,, \quad (6.13)$$

where $\mathcal{G}_4(p^2,\lambda^2)$, $\lambda^2 \neq 0$, is now both ultraviolet and infrared finite . Finally this diagram contributes to Z_3 with

$$Z_3^{(4)} = \frac{i(4\pi)^2}{2}\left(\frac{1}{2}I^{(3)}_{\log}(\lambda^2) + I^{(2)}_{\log}(\lambda^2)\right). \quad (6.14)$$

The diagram displayed in figure 5 is easy to evaluate. It also contains two one-loop subdiagrams just as the diagram in figure 3. Therefore we expect that it shall contribute to Z_3 with a term proportional to $I^{(3)}_{\log}(\lambda^2)$ as well. For the sake of completeness we write its contribution to (6.1):

$$H_5 = \frac{i(4\pi)^6}{4}\int_q \frac{1}{q^2(p+q)^2}\int_l \frac{1}{l^2(q+l)^2}\int_k \frac{1}{k^2(k+q)^2}\,. \quad (6.15)$$

Notice that the integrals over l and k in the equation above represent the one loop subdiagrams. We proceed in a similar fashion as we did for the diagram in figure 3 to obtain

$$\widetilde{H}_5(p^2) = \frac{-i(4\pi)^2}{4} I^{(3)}_{\log}(\lambda^2) + \mathcal{G}_5(p^2,\lambda^2)\,, \quad (6.16)$$

where the notation is now obvious. Hence, the renormalization constant Z_3 should contain the following contribution,

$$Z_3^{(5)} = \frac{i(4\pi)^2}{4} I^{(3)}_{\log}(\lambda^2)\,, \quad (6.17)$$

in order to cancel the divergence that stemmed from this diagram.

Finally we study the diagram in figure 6. In opposite to the previous three loop diagrams, it contains no subdiagrams at all. Its contribution can be cast as [26]:

$$H_6 = \frac{i(4\pi)^6}{4}\int_{l,q,k} \frac{1}{q^2k^2(p-l)^2(l-q)^2(l-k)^2(q-k)^2}\,. \quad (6.18)$$

An irreducible loop integral can be displayed after expanding the propagator that contains p in H_6 according to (2.1) to obtain

$$H_6 = \frac{i(4\pi)^6}{4}\int_{l,q,k} \frac{1}{q^2k^2l^2(l-q)^2(l-k)^2(q-k)^2} + \mathcal{F}_6(p^2) \equiv H_6^{\infty} + \mathcal{F}_6(p^2)\,, \quad (6.19)$$

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$\mathcal{F}_6(p^2)$ being a ultraviolet finite piece as usual. We show in appendix B that

$$H_6 = -\frac{3}{2} i (4\pi)^6 \zeta(3) I_{\log}(\lambda^2) + \mathcal{G}_6(p^2, \lambda^2) \,, \tag{6.20}$$

where $\zeta(x)$ stands for the Riemann zeta-function. Therefore we define the last contribution to Z_3,

$$Z_3^{(6)} = \frac{3}{2} i (4\pi)^6 \zeta(3) I_{\log}(\lambda^2) \,, \tag{6.21}$$

to finally write

$$Z_3 = Z_3^{(3)} + \cdots + Z_3^{(6)} \,. \tag{6.22}$$

7. The β-function

In [22] we have compared implicit, dimensional, differential and BPHZ renormalization and shown, within these intrinsically distinct frameworks, how renormalizations schemes and scales are related. It goes as follows. Minimally subtracting the infinities in dimensional regularization (i.e. removing only the poles) delivers a finite (non-counterterm) term which depends upon the renormalization scale μ. It results that such term is the same that appears should we employ differential renormalization, except for a simple rescaling of its typical arbitrary scale M^2 (that is to say, a finite counterterm). The latter is the scale of a Callan-Symanzik renormalization group equation satisfied by the vertex function [8]. A minimal renormalization scheme within implicit regularization corresponds to subtracting the irreducible loop integrals in a mass independent fashion through relations like (4.6), (5.8) and (6.9) where an arbitrary mass scale λ is introduced. It turns out that the resulting finite part can be identified with the one from differential renormalization (Fourier transformed to momentum space) after a simple rescaling of λ. The difference is that in our framework we do have counterterms from which we may calculate the renormalization group functions. Moreover our renormalized amplitude also satisfies a Callan-Symanzik renormalization group equation governed by the arbitrary scale λ.

An important non-trivial test of our framework is to calculate the β-function of the WZ model which is well known to four loop order [37, 38]. We start with equation (3.5). Because g_0 cannot depend upon our arbitrary scale λ we are led to

$$\beta = \frac{3g}{2} \lambda \frac{\partial Z_\phi}{\partial \lambda} \equiv \frac{3g}{2} \dot{Z}_\phi \,, \tag{7.1}$$

where the dot stands for the logarithmic derivative, i.e. $\dot{A} \equiv \lambda(\partial/\partial\lambda)A$. Now we revoke (3.6) to write a coupling constant expansion for the β function, viz.

$$\begin{aligned}\frac{\beta}{6\pi} = {} & \dot{Z}_1 \left(\frac{g}{4\pi}\right)^3 + (\dot{Z}_2 + 2Z_1\dot{Z}_1)\left(\frac{g}{4\pi}\right)^5 + \\ & + \left(\dot{Z}_3 + 5\dot{Z}_1 Z_2 + 2Z_1\dot{Z}_2 + 4\dot{Z}_1 Z_1^2\right)\left(\frac{g}{4\pi}\right)^7 + \\ & + \cdots \,. \end{aligned} \tag{7.2}$$

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Using our results displayed in equations (4.8), (5.9) and (6.22) and that

$$\begin{aligned}\dot{I}_{\log}(\lambda^2) &= -2b\,, \\ \dot{I}^{(2)}_{\log}(\lambda^2) &= 2I_{\log}(\lambda^2) - 2b \qquad \text{and} \\ \dot{I}^{(3)}_{\log}(\lambda^2) &= 4I^{(2)}_{\log}(\lambda^2) - 4b\,,\end{aligned} \tag{7.3}$$

we get

$$\beta = g\left[\frac{3}{2}\left(\frac{g}{4\pi}\right)^2 - \frac{3}{2}\left(\frac{g}{4\pi}\right)^4 + \left(\frac{21}{4} + \frac{9}{2}\zeta(3)\right)\left(\frac{g}{4\pi}\right)^6\right]. \tag{7.4}$$

Some comments are in order. Whilst the one and two-loop coefficients of the β function are scheme-independent as opposed to the three loop coefficient, the coefficient of $\zeta(3)$ should be universal. That is because the diagram from which such coefficient stems is primitive.

Our results for the β-function agree with the calculation performed in [37, 38]. In contrast, the coefficient we obtain for $\zeta(3)$ is different from the one obtained in the original calculation in DR [26]. It is also different from the result obtained in [30] which extends [26] to four loop order. In fact the coefficient of $\zeta(3)$ obtained in [30] agrees with the calculation in DfR [31]. We believe that the result displayed by us as well as in [37] and [38] is correct because in [38] their exact result is consistent with neither β nor γ receiving corrections beyond one loop in the $N = 2$ case. We attribute the difference between our results and those displayed in [26, 30] and [31] to an overall (symmetry) factor associated to the diagram that represents $\mathcal{H}_6$ which should be $1/4$. Except for this our results agree with the calculation in DfR. In particular we see that our three loop scheme-dependent coefficient corresponds to a momentum space subtraction (MOM) in which a subtraction is performed at $p^2 = \lambda^2 \neq 0$ [26]. This comparison clearly shows the connection between IR, DfR and DR in what concerns the definition of the renormalization scheme. This is interesting because the *modus operandi* of such schemes are quite distinct.

8. Conclusions and perspectives

If on one hand at the near future experiments at the LHC or at a linear e^+e^- collider will test decisively supersymmetric extensions of the standard model, on the other hand the theoretical machinery which exploits such predictions must have a thorough control upon the regularization and renormalization of supersymmetric Yang-Mills theories.

Since complete regularization framework which is both gauge and supersymmetric invariant has yet not been constructed, it is reasonable to exploit a framework which is gauge invariant and works in the physical dimension of the theory. This work is a fundamental step in this direction which should help to address important issues such as:

1. gauge field theories with soft supersymmetry-breaking terms: for instance QCD with soft breaking which has a particular phenomenological interest. In [32] models with soft-breaking terms have been studied in the Wess-Zumino gauge. However there appears new parameters which have no clear interpretation as being either susy or soft breaking terms (see however [5]). IR should be a useful tool to renormalize softly

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broken susy gauge theories avoiding some complications used in dimensional methods such as the introduction of Slavnov-Taylor identities as constraint equations in order to ensure both gauge invariance and supersymmetry [33].

2. the anomaly puzzle: the axial and energy momentum trace anomalies seem to lead to the conclusion that the β function of supersymmetric gauge field theories should be exhausted by the first loop correction [34]. Whilst this appear to be the case in models with $N = 2$ supersymmetry, the case $N = 1$ is less clear [35] particularly when one uses dimensional regularization methods. In DfR [36], which like IR does not resort to analytic continuation on the space time dimension, the infrared origin of the two loop coefficient of the β-function of $N = 1$ SYM has been discussed (see also [39]). IR, which operates in momentum space and on the physical dimension of the theory, clearly isolates infrared and ultraviolet divergences by means of distinct scales. Thus we expect that IR can shed some light on the infrared effects and scheme dependence of the higher order corrections (if any) to the β-function of $N = 1$ supersymmetric gauge theories as well as establish a correspondence with other frameworks such as the canonical wilsonian coupling flow [40].

A. Finite part of $G(p^2)$

We can easily obtain the finite part of $G(p^2)$ in a simple form using Rosner's technique of expanding propagators in Chebyshev's polynomials [**?**].

In Euclidean space (5.4) reads:

$$\widetilde{G}_{\mathrm{E}}(p^2) = -(4\pi)^2 \int_{q_E} \frac{1}{q^2(p+q)^2} \ln\left(\frac{\lambda^2 e^2}{q^2}\right) . \tag{A.1}$$

Because $\widetilde{G}_E(p^2)$ is a function of p^2 only we may average over the directions of p in four dimensions. Let C_m be the Chebyshev's polynomial of order m. It satisfies a four dimensional version of its orthogonality relation, viz.

$$\int \frac{d\Omega_p}{2\pi^2} C_n\left(\frac{p\cdot q}{pq}\right) C_m\left(\frac{p\cdot q}{pq}\right) = \delta_{mn} , \tag{A.2}$$

with $C_0 = 1$. Following [**?**] we write

$$\frac{1}{(p+q)^2} = \frac{1}{pq}\sum_n \langle p|q\rangle^{n+1} C_n\left(\frac{p\cdot q}{pq}\right) , \tag{A.3}$$

where

$$\langle p|q\rangle = \begin{cases} \dfrac{p}{q} & \text{if } p < q \\ \dfrac{q}{p} & \text{if } p > q \end{cases} . \tag{A.4}$$

Using (A.3) and (A.2) in (A.1) yields

$$\widetilde{G}_{\mathrm{E}}(p^2) = -(4\pi)^2 \int_{q_E} \frac{1}{q^3 p} \langle q|p\rangle \ln\left(\frac{\lambda^2 e^2}{q^2}\right) , \tag{A.5}$$

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$\int_{q_E} \equiv 1/(2\pi)^4 \int d\Omega_q \int q^3\, dq$. After integrating over the angles we get

$$\widetilde{G}_E(p^2) = -\ln\left(\frac{\lambda^2 e^2}{p^2}\right) - 1 - \int_{p^2}^{\infty} \frac{1}{x} \ln\left(\frac{\lambda^2 e^2}{x}\right)\, dx\,. \tag{A.6}$$

Take the last term in the equation above. That integral contains the ultraviolet divergent piece from which we define the renormalization constant Z_2. Moreover as we discussed earlier it should be infrared safe. To see that explicitly let us split the integral from p^2 to ∞ in three pieces: from p^2 to $\lambda^2 \neq 0$ plus from λ^2 to μ^2 (infrared cutoff) plus a remaining contribution from μ^2 to ∞. The latter can be easily shown to be related to (5.7),

$$\int_{\mu^2}^{\infty} \frac{1}{x} \ln\left(\frac{\lambda^2 e^2}{x}\right)\, dx = \frac{1}{b} I_{\log}^{(2)}(\mu^2) + 1\,, \tag{A.7}$$

whereas the two other pieces added together yield $1/2\{\ln^2[p^2/(\lambda^2 e^2)] - \ln^2[\mu^2/(\lambda^2 e^2)]\}$. Note that such contribution diverges as $\mu \to 0$. However this behaviour is tamed if we correctly define a genuine ultraviolet divergent object by exchanging μ against $\lambda \neq 0$ with the help of equation (5.8). Putting all the results together we arrive at

$$\widetilde{G}_{\mathrm{E}}(p^2) = -\ln\left(\frac{\lambda^2 e^2}{p^2}\right) - \frac{1}{2}\ln^2\left(\frac{\lambda^2 e^2}{p^2}\right) - \frac{I_{\log}^{(2)}(\lambda^2)}{b}\,, \tag{A.8}$$

from which we may finally define the finite part of the two loop amplitude which in turn satisfies a Callan-Symanzik renormalization group equation with renormalization scale λ. In Minkowski space it reads:

$$\widetilde{G}_{\mathrm{fin}}(p^2) = -\ln\left(\frac{-\lambda^2 e^2}{p^2}\right) - \frac{1}{2}\ln^2\left(\frac{-\lambda^2 e^2}{p^2}\right). \tag{A.9}$$

B. Divergent structure of $H_6(p^2)$

The graph displayed in figure 6 contains no subdiagrams. The irreducible loop integral that represents the ultraviolet divergent content of this graph can be displayed by studying H_6^∞ in equation (6.19) . It will be convenient to express the corresponding counterterm as one of the basic divergent integrals which have appeared so far, namely $I_{\log}(\lambda^2)$, $I_{\log}^{(2)}(\lambda^2)$, $I_{\log}^{(3)}(\lambda^2)$, etc. Because this diagram is primitive and ultraviolet logarithmically divergent, Z_3^6 is expected to be proportional do $I_{\log}(\lambda^2)$. We use the same technique of appendix A in order to extract such ultraviolet behaviour. Moreover, it will become clear why the coefficient of $\zeta(3)$ in (6.21), which will appear in the three loop contribution to the β function, is universal (save a coupling constant redefinition involving $\zeta(3)$) despite the β-function being scheme dependent at the three loop level.

We temporarily work in Euclidean space where H_6^∞ reads

$$H_{6\mathrm{E}}^\infty = \frac{(4\pi)^6}{2} \int_{l,q,k} \frac{1}{q^2 k^2 l^2 (l-q)^2 (l-k)^2 (q-k)^2}\,. \tag{B.1}$$

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Equations (A.2) and (A.3) enable us to write

$$H_{6\mathrm{E}}^{\infty} = 2(4\pi)^2 \int_l \frac{1}{l^4}\, \mathcal{Y} \tag{B.2}$$

where

$$\mathcal{Y} = \int \frac{d\Omega_q}{2\pi^2} \int \frac{d\Omega_k}{2\pi^2} \frac{1}{q\ k} \sum_{m,n,p} \langle l|q\rangle^{n+1} \langle l|k\rangle^{m+1} \langle q|k\rangle^{p+1} C_n(l,q)\, C_m(l,k)\, C_p(q,k)\,. \tag{B.3}$$

Thus the identity

$$\int \frac{d\Omega_k}{2\pi^2} C_n\left(\frac{l\cdot k}{lk}\right) C_m\left(\frac{q\cdot k}{qk}\right) = \frac{\delta_{mn}}{n+1} C_n\left(\frac{l\cdot q}{lq}\right), \tag{B.4}$$

applied to (B.3) yields, after straightforward algebra,

$$\mathcal{Y} = \frac{3}{2} \sum_{n=0}^{\infty} \frac{1}{(n+1)^3} = \frac{3}{2}\zeta(3)\,. \tag{B.5}$$

Therefore, in Minkowki space, after introducing an infrared cutoff μ and making use of equation (4.6) we obtain

$$H_6^{\infty} = -3i(4\pi)^2\zeta(3) I_{\log}(\lambda^2) + 3\zeta(3)\ln\left(\frac{\lambda^2}{\mu^2}\right), \tag{B.6}$$

where $\lim_{\mu\to 0}[3\zeta(3)\ln(\lambda^2/\mu^2) + \mathcal{F}_6(p^2,\mu^2)] = \mathcal{G}_6(p^2,\lambda^2)$ is well-defined.

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